EFFECT OF HIGH-FREQUENCY ELECTRIC FIELD ON DISPERSION OF SURFACE WAVES EXCITED BY AN ELECTRON BEAM

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In review article [1] considerable space has been devoted to the investigation of the effect of stabilization of different type of instabilities by high-frequency fields. The stabilization of hydrodynamic current instability by a homogeneous high-frequency electric field was first investigated in [2].

§1. We assume that the region x > 0 is occupied by a cold homogeneous isotropic plasma with density n_0, whose electrons move on the background of stationary ions with velocity $\mathbf{u}$ parallel to the external electric field $\mathbf{E}_{ext} = \mathbf{E}_0 \sin(\omega_0 t)$. The direction of vector $\mathbf{E}_0$ coincides with the z axis. The plasma–vacuum interface (x = 0) is assumed to be sharp.

Following the method of separation discussed in [3] in connection with the investigation of parametric resonance in a cold inhomogeneous plasma, it can be shown that the solution of the problem of excitation of surface waves by an electron current in the presence of a high-frequency field can be separated into the solution of the spatial (not depending on the amplitude of the high-frequency field) and temporal (intrinsically parametric) parts. The values of the separation constant p are determined from the solution of the spatial part of the problem. The system of temporal equations of renormalized plasma frequencies $\omega_{pe}^2 \to p^2$, $\omega_{pi}^2 \to (m_e/m_i)p^2$ reduces to the system of differential equations derived in [2]; if the frequency of the field is larger than the characteristic frequency of the plasma, this system has the following form:

$$\frac{\partial^2}{\partial t^2}\langle\nu_{e1}\rangle + p^2\left[\langle\nu_{e1}\rangle + \langle\nu_{i1}\rangle e^{ik_z ut} J_0(a)\right] = 0; \tag{1.1}$$

$$\frac{\partial^2}{\partial t^2}\langle\nu_{i1}\rangle + \frac{m_e}{m_i} p^2\left[\langle\nu_{i1}\rangle + \langle\nu_{e1}\rangle e^{-ik_z ut} J_0(a)\right] = 0, \tag{1.2}$$

where $\langle\nu_{\alpha1}\rangle = \frac{\omega_0}{2\pi}\int\limits_0^{2\pi/\omega_0} \nu_{\alpha1}dt$; $\nu_{\alpha1}$ is the temporal part of the density perturbation of the particles of type α in the coordinate system oscillating with the particles of the given type. The nonequilibrium quantities are expressed in the form $f(\mathbf{r}, t) = f_1(t)f_2(\mathbf{r})$; $J_0(a)$ is a Bessel function.

For the investigated geometry the equation describing the spatial part of the problem is written in the following way:

$$\frac{d}{dx}\left[\varepsilon\frac{d\varphi_2}{dx}\right] - k_z^2\varepsilon\varphi_2 = 0, \tag{1.3}$$

where $\varepsilon = 1 - \omega_{pe}^2/p^2$.

Determining the spatial part of the electric field potential φ_2 in the regions $x \lessgtr 0$ from Eq. (1.3) and using the continuity conditions for the quantities φ_2 and $\varepsilon d\varphi_2/dx$ at the point x = 0 we find the separation constant

$$p \equiv \overset{*}{\omega}_{pe} = \omega_{pe}/\sqrt{2}.$$

The dispersion equation, describing the excitation of surface waves by the electric current, follows from the system of equations (1.1), (1.2):

$$\left[(\omega - k_z u)^2 - \overset{*}{\omega}{}_{pe}^2\right](\omega^2 - \overset{*}{\omega}{}_{pi}^2) = \overset{*}{\omega}{}_{pe}^2\overset{*}{\omega}{}_{pi}^2 J_0^2(a). \tag{1.4}$$

In the nonresonance case, when $k_z u$ is not close to $\overset{*}{\omega}_{pe}$, from Eq. (1.4) we obtain the expression for the frequency of oscillations:

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$$\omega^2 = \omega_{pi}^{*2}\left[(k_z u)^2 - \omega_{pe}^{*2}\left(1 - J_0^2(a)\right)\right]\left((k_z u)^2 - \omega_{pe}^{*2}\right)^{-1}. \tag{1.5}$$

According to formula (1.5) the growing solutions are possible when the following inequalities are satisfied:

$$\omega_{pe}^{*}\left(1 - J_0^2(a)\right)^{1/2} < k_z u < \omega_{pe}^{*}. \tag{1.6}$$

In the absence of the high-frequency field the left-hand side of the inequality vanishes and condition (1.6) coincides with the condition of development of Buneman instability. The difference of the left-hand side of the inequality (1.6) from zero, determined by the amplitude of the high-frequency field, denotes the narrowing of the instability region.

As is well known, the maximum growth increment of oscillations is obtained when the condition $k_z u \approx \omega_{pe}^*$ is satisfied. In this case the frequency and the growth increment of the surface waves are given by the expressions

$$\omega = 2^{-4/3}\left(\frac{m_e}{m_i}\right)^{1/3} J_0^{2/3}(a)\,\omega_{pe}^{*}; \tag{1.7}$$

$$\gamma = 3^{1/2} 2^{-4/3}\left(\frac{m_e}{m_i}\right)^{1/3} J_0^{2/3}\omega_{pe}^{*}. \tag{1.8}$$

It follows from (1.7), (1.8) that in the presence of the high-frequency field the frequency ω and the increment of the unstable oscillations γ decerase by a factor of $J_0^{2/3}(a)$.

§2. Let us consider the excitation of surface waves by a monoenergetic beam of cold electrons moving with velocity u_b along the interface ($x = 0$) of the beam and the homogeneous isotropic plasma at rest at the presence of the high-frequency field $\mathbf{E}_{ext} = \mathbf{E}_0 \sin(\omega_0 t)$. The direction of vectors u_b and E_0 coincides with the z axis.

For the values of the pumping frequency ω_0 appreciably larger than the characteristic frequency of the plasma, we find the equations for the functions $\nu_{e_1}(t)$, $\nu_{i_1}(t)$, $\nu_{b_1}(t)$ averaged over a period of the high-frequency field by the method of separation:

$$\frac{\partial^2}{\partial t^2}\langle \nu_{e1}\rangle + p^2\left[\langle \nu_{e1}\rangle + \langle \nu_{i1}\rangle J_0(a) + \langle \nu_{b1} e^{-ik_z u_b t}\rangle\right] = 0; \tag{2.1}$$

$$\frac{\partial^2}{\partial t^2}\langle \nu_{i1}\rangle + \frac{m_e}{m_i} p^2\left[\langle \nu_{i1}\rangle + \langle \nu_{e1}\rangle J_0(a) + \left\langle \nu_{b1} e^{-ik_z u_b t}\right\rangle J_0(a)\right] = 0; \tag{2.2}$$

$$\frac{\partial^2}{\partial t^2}\langle \nu_{b1}\rangle + \alpha p^2\left[\langle \nu_{b1}\rangle + \langle \nu_{e1} e^{ik_z u_b t}\rangle + \langle \nu_{i1} e^{ik_z u_b t}\rangle J_0(a)\right] = 0, \tag{2.3}$$

where ν_{b_1} is the temporal part of the perturbation of the beam density; $\alpha \equiv n_b/n_0 \ll 1$; n_b and n_0 are the equilibrium densities of the beam and the plasma, respectively.

From Eqs. (2.1)-(2.3) we obtain the dispersion equation determining the frequency of surface waves excited by the beam in the presence of the high-frequency field:

$$\left(\omega^2 - \omega_{LF}^2\right)\left[(\omega - k_z u_b)^2\left(\omega^2 - \omega_{HF}^2\right) - \alpha\omega^2\omega_{pe}^{*2}\right] = 0, \tag{2.4}$$

where

$$\omega_{LF}^2 = \omega_{pi}^{*2}\left(1 - J_0^2(a)\right); \quad \omega_{LF}^2 = \omega_{pe}^{*2} + \omega_{pi}^{*2} J_0^2(a).$$

In the nonresonance case, when $k_z u_b$ is not close to ω_{HF}, from Eq. (2.4) we determine the condition of pumping of surface waves:

$$0 < k_z u_b < \omega_{HF}. \tag{2.5}$$

In the resonance case ($k_z u_b \approx \omega_{HF}$) the frequency of the unstable surface waves is written in the form $\omega = \omega_{HF} + \Delta$, where

$$\operatorname{Re}\Delta = -2^{-4/3}\left(\omega_{pe}^{*2} k_z u_b\right)^{1/3}\alpha^{1/3}; \tag{2.6}$$

$$\operatorname{Im}\Delta \equiv \gamma = 3^{1/2}\, 2^{-4/3}\left(\omega_{pe}^{*2} k_z u_b\right)^{1/3}\alpha^{1/3}. \tag{2.7}$$

It follows from Eqs. (2.5)-(2.7) that the high-frequency field has no significant effect on the nature of excitation of the surface waves by a monoenergetic beam of electrons, insignificantly extending the region of the values of the wave vectors of unstable waves compared to the case when there is no pumping field.

In conclusion, we discuss a possible practical application of the theoretical results obtained here. The excitation of surface waves by beams of nonrelativistic electrons in a cold plasma in the presence of a variable electric field is investigated. In certain conditions the interaction of the electron beam with plasma leads to the excitation of unstable oscillations when initially small perturbations of density and particle velocities, the amplitudes of the self-consistent electric field in the plasma, and so forth, exponentially increase with time (or with the increase of the spatial coordinate). However, a need arises for the suppression of the beam instability; this occurs in the operation of experimental equipment in which electron beams exist. Among such devices we can mention discharges with longitudinal electric field (for example, installations of "tokamak" type). On the periphery of the plasma configuration the density of the plasma particles is considerably smaller than the plasma density at the axis of the system. For example, in the case of a sufficiently rapid process when skinning of the current occurs, the plasma density has a sharp discontinuity at the boundary of the discharge. Under the action of the rotating electric field the plasma electrons in the peripheral region of the discharge can get accelerated considerably faster than in the central region and may go over into the "escape" regime. Hence, a plasma configuration with a monoenergetic beam of electrons develops exciting surface waves in a resonance manner. The development of such instabilities can significantly change the equilibrium configuration of the plasma. One of the possible methods of stabilization of unstable surface waves by a high-frequency field is proposed in this work.

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LIMITING CURRENT OF A CONICAL BEAM

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The study of the motion of a charged particle (electron) in the magnetic field of an electron beam is of great practical and theoretical interest. For example, Alfvén [1] studied the motion of a test electron in the field of a cylindrical beam and showed that there is a critical Alfvén current in a neutral beam. For currents above the limiting value [$I_A = (mc^3/e)\gamma\beta \approx 17\gamma\beta$ kA] an electron can reach a certain point and turn back and not move along the beam.

We investigate the motion of a particle in the magnetic field of a conical relativistic electron beam to determine the effect of divergence on the existence of a limiting current. There is no electric field in the beam since it is assumed that there is a compensating background of positive stationary ions.

The equation for the magnetic field of a conical beam emitted from a center is

$$\operatorname{rot} \mathbf{H} = (4\pi/c)\mathbf{j},$$

where $\mathbf{j} = I_0\mathbf{r}/r^3$ and $I_0 = \text{const}$. Clearly the field can be described by a single component $\mathbf{H} = H_\varphi \mathbf{e}_\varphi$, where $\mathbf{e}_\varphi$ is a unit vector in the direction of variation of the azimuthal angle φ of a spherical coordinate system r, θ, φ. In this case

$$H = H_\varphi|_{\theta\leqslant\theta_0} = \frac{4\pi I_0}{cr}\operatorname{tg}\frac{\theta}{2}. \quad (1)$$

It is assumed that there is a current only for $\theta \leq \theta_0$. Outside the cone the magnetic field is

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